Electron channel drop tunnelling

1999 J. Phys.: Condens. Matter 11 L247

(http://iopscience.iop.org/0953-8984/11/23/102)

J. Phys.: Condens. Matter **11** (1999) L247–L252. Printed in the UK PII: S0953-8984(99)01703-8

LETTER TO THE EDITOR

Electron channel drop tunnelling

L Dobrzynski, B Djafari-Rouhani, A Akjouj, J O Vasseur† and M L H Lahlaouti‡

Laboratoire de Dynamique et Structures des Matériaux Moléculaires, UPRESA CNRS 8024, UFR de Physique, Université de Lille 1, 59655 Villeneuve d'Ascq Cédex, France

Received 9 February 1999, in final form 20 April 1999

Abstract. We consider the tunnelling between two monomode quantum wires. We give, in closed form, the conditions for selective transfer of a single propagating electron from one wire to the other, leaving all the neighbouring states unaffected. We illustrate the results of the analysis by analytical solutions for a simple composite system made out of GaAs wires. The electron channel drop tunnelling in this system is due to one localized state situated within a gap of the coupling device. The energy of this state is tuned to be that of the Fermi energy.

Electron waveguiding in quantum wires was first identified in quantum point contacts at low temperatures where the conductance was found to be quantized in discrete values of $2e^2/h$ [1, 2]. Electron waveguiding has also been observed during the transfer of states between electron guides [3, 4]. Such transfer processes are particularly important for single-energy electron spectroscopy and electron directional couplers. Of special interest is the selective transfer of a monoenergy electron from one quantum wire to the other, leaving all other states unaffected. However, to our knowledge, the general conditions needed to realize optimal electron transfer have not been established until now.

In this letter we give first, in closed form, conditions for complete channel drop tunnelling for any monomode electronic coupling device having the symmetry of two orthogonal mirror planes. We then illustrate these results using a simple device made from monomode GaAs quantum wires as an example.

Let us consider the generic system schematically presented in figure 1(a). This system has the symmetry of two mirror planes, which are shown by dashed lines. The two continuums are the two infinite wires passing by points (1, 2) and (3, 4), respectively. It is also convenient to consider the finite system (1, 2, 3, 4) obtained by removing the four semi-infinite lines at points 1, 2, 3 and 4. Consider a propagating state excited in the semi-infinite line attached to point 1. The corresponding reflection R and transmission coefficients T_{1j}, $j = 2, 3, 4$ are related to the elements $g(1j)$ of the Green's function [5] of this system by the relations

$$R = |1 + 2\mathrm{i}\alpha\ g(11)|^2 \tag{1a}$$

$$T_{1j} = |2\mathrm{i}\alpha\ g(1j)|^2 \qquad j = 2, 3, 4. \tag{1b}$$

In equation (1), the parameter α is defined, for monomode electrons, as $\alpha = \sqrt{2mE}/\hbar$, where E is the electron energy, m its mass and $\mathrm{i} = \sqrt{-1}$. By simple symmetry arguments, it is

† To whom correspondence should be addressed.

‡ Permanent address: Département de Physique, Faculté des Sciences, BP 2121, Université A Essaadi, Tetouan, Morocco.

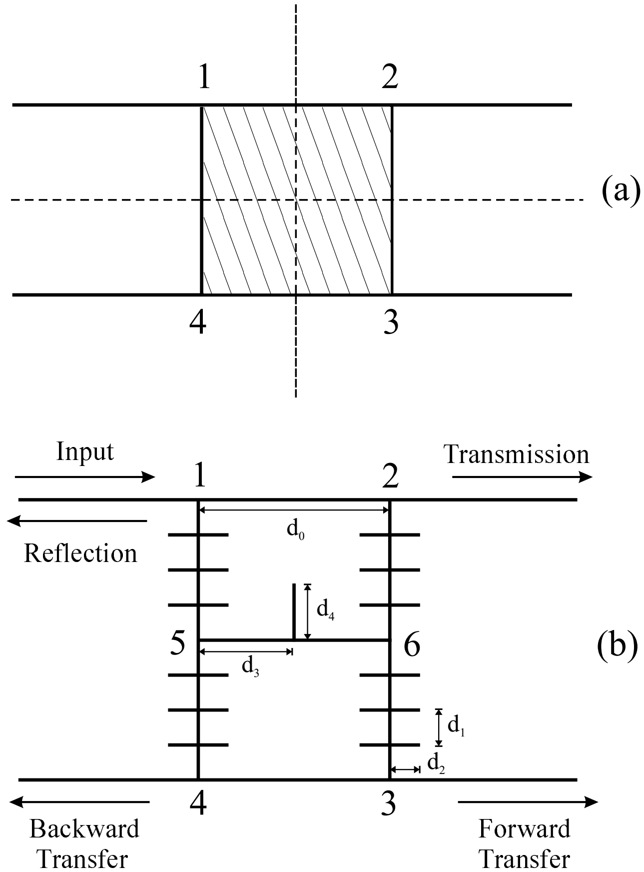

Figure 1. (a) The general system under consideration. (b) The special system for application.

straightforward to show that the elements $g(1j)$ must have the following general form

$$g(11) = Z_1 + Z_2 + Z_3 + Z_4 \tag{2a}$$

$$g(12) = Z_1 + Z_2 - Z_3 - Z_4 \tag{2b}$$

$$g(13) = Z_1 - Z_2 + Z_3 - Z_4 \tag{2c}$$

$$g(14) = Z_1 - Z_2 - Z_3 + Z_4 \tag{2d}$$

where

$$Z_n = 1/[4\alpha(y_n - \mathrm{i})] \qquad n = 1, 2, 3, 4. \tag{3}$$

Equations (1), (2) and (3) are valid for any composite system having the symmetry of two mirror planes. In equation (3), the y_n are purely real quantities determined by the finite structure contained in the shaded square (1, 2, 3, 4), and the imaginary terms are due to the semi-infinite wires. Combining equations (1), (2) and (3), one finds easily that in order to have a complete transfer of a propagating state at a given energy, E_0, from site 1 to site 3 (namely $T_{13} = 1$ and $R = T_{12} = T_{14} = 0$), one must fulfill, for $E = E_0$, the following conditions

$$y_1 = y_3 = -1/y_2 = -1/y_4. \tag{4}$$

If one wants this energy E_0 to be in the middle of a gap ΔE for which only direct transmission exists (namely $T_{12} = 1$ and $R = T_{13} = T_{14} = 0$), one must fulfill the following conditions in this domain

$$y_1 = y_2 = -1/y_3 = -1/y_4. \tag{5}$$

In other words, these conditions (5) require that the system which couples the two continuums must have a gap in the above defined energy domain. The conditions (4) imply that the complete system must have one resonant energy E_0 inside this gap. The conditions (4) and (5) enable one to determine the parameters of the system, provided the y_n are known. Detailed expressions of the y_n will be given later for a particular system having a specific geometry.

In this letter, we consider mesoscopic structures made of one-dimensional quantum wires. As already observed by Singha Deo and Jayannavar [6], this one-channel case provides a good approximation to a real wire with finite width at low temperatures, at which only the lower subband is filled. Moreover, energy-level spacings produced by transverse confinement must be larger than the energy range of the longitudinal transport and thermal broadening, $k_B T$. In this regime, a quantum wire behaves as a single-mode electron waveguide. In such mesoscopic systems, electrons traverse the device as coherent waves with negligible inelastic scattering [7] and electron transport is then identical to microwave propagation through waveguides.

With these assumptions, the electronic currents I_j for coherent transport from contact 1 to contact $j = (2, 3, 4)$ are given by the Landauer–Büttiker formula [8]

$$I_j = \frac{2e}{h}\int_0^\infty T_{1j}(E)\Big[f_j(E) - f_1(E)\Big]\mathrm{d}E \tag{6}$$

where

$$f_j(E) = \Big[1 + \exp\Big((E - \mu_j)/k_B T\Big)\Big]^{-1}$$

is the Fermi function for contact j and the electrochemical potentials μ_j are chosen such that $\mu_2 = \mu_3 = \mu_4 < \mu_1$. For $k_B T \ll (\mu_1 - \mu_2)$, equation (6) reduces to

$$I_j = \frac{2e}{h}T_{1j}(E_F)(\mu_1 - \mu_2) \tag{7}$$

where E_F is the Fermi energy. In these conditions, the electron current will flow from terminal 1 to terminal 2 in all situations, with one exception. When the coupling device is tuned so that $E_0 = E_F = (\mu_1 + \mu_2)/2$, the electron current will flow from terminal 1 to terminal 3. This system therefore acts as an electron directional coupler.

In what follows, we illustrate the general conditions (4) and (5) by choosing one very simple system, shown schematically in figure 1(b). This system is built from the two infinite quantum wires passing by the points (1, 2) and (3, 4), respectively. The distance between points 1 and 2, d_0, is the same as that between points 3 and 4. Four identical monomode structures are branched between points (1, 5), (5, 4), (2, 6) and (6, 3). These structures have N equidistant (distance d_1) sites. Stars of N' side branches of length d_2 are grafted onto the $(N - 2)$ internal sites. In figure 1(b), N and N' are equal to 5 and 2, respectively. Similar structures have been studied before [6], and have been shown [9] to have giant gaps. They also allow the adjustment of the energy range of these gaps to a desired domain by tuning the distance d_2, d_1 and the numbers N and N'. One wire of length $2d_3$ is fixed between points 5 and 6 with a side branch of length d_4 in its middle. Following earlier results [9], it is easy to obtain the elements of the Green's function of such structures.

In order to give, for this special system, the analytical expressions of the y_n defined in equation (3), let us define the quantities

$$A_m = -1/\tan(\alpha d_m) \tag{8}$$

$$B_m = 1/\sin(\alpha d_m) \tag{9}$$

with $m = 0, 1, 2, 3, 4$. A_0 and B_0 are associated with the finite parts situated between points (1, 2) and (3, 4) of the infinite wires. We also define the terms A_5 and B_5 related to the structures with large gaps grafted between points (1, 5), (5, 4), (2, 6) and (6, 3) as

$$A_5 = -N' A_2 - A_1 - B_1 \sin(Nkd_1)/\sin[(N-1)kd_1] \tag{10}$$

$$B_5 = B_1 \sin(kd_1)/\sin[(N-1)kd_1] \tag{11}$$

where k is defined by

$$\cos(kd_1) = -\frac{1}{B_1}\left(A_1 + \frac{N'}{2}A_2\right). \tag{12}$$

Equation (12) is the dispersion relation [9] of such an infinite guide.

The properties of the structure grafted between points 5 and 6 in figure 1(b) are related to

$$B_6 = -B_3^2/(2A_3 + A_4) \tag{13}$$

$$A_6 = A_3 + B_6. \tag{14}$$

The definition of these quantities leads to the following expressions for the y_n associated with the final system depicted in figure 1(b). They are given as

$$y_1 = y_2 - 2B_5^2/(2A_5 + A_6 + B_6) \tag{15}$$

with

$$y_2 = A_0 + B_0 + A_5 \tag{16}$$

$$y_3 = A_0 - B_0 + A_5 \tag{17}$$

$$y_4 = y_3 - 2B_5^2/(2A_5 + A_6 - B_6). \tag{18}$$

Now, for $k_B T \ll (\mu_1 - \mu_2)$, we are able to accurately determine these system parameters for a complete channel drop tunnelling between the two continuums at an energy $E_0 = E_F = (\mu_1 + \mu_2)/2$ falling in the middle of the gap ΔE. First, the condition $y_2 y_3 = -1$ (equations (4) and (5)) is satisfied for

$$A_5(E_0) = 0. \tag{19}$$

We determine the length d_0 such that

$$y_2(E_0) = -y_3(E_0) = 1 \tag{20}$$

by choosing

$$\tan(\alpha_0 d_0/2) = 1 \tag{21}$$

where $\alpha_0 = \sqrt{2mE_0}/\hbar$.

So we study the quantity $A_5(E)$ given by equation (10) and choose N, N', d_1 and d_2 in order that equation (19) shall be satisfied inside the ΔE gap. The quality factor of the transferred energy peak increases when the gap ΔE increases, as well as the requirements on the precision of the distances d_m. The remainder of the conditions given by equation (4) are then satisfied for

$$A_6(E_0) = 0 \tag{22}$$

$$B_6(E_0) = B_5^2(E_0). \tag{23}$$

These last two conditions lead to

$$\tan(\alpha_0 d_3) = 1/B_5^2(\alpha_0) \tag{24}$$

$$\tan(\alpha_0 d_4) = -\frac{1}{2}\tan(2\alpha_0 d_3) \tag{25}$$

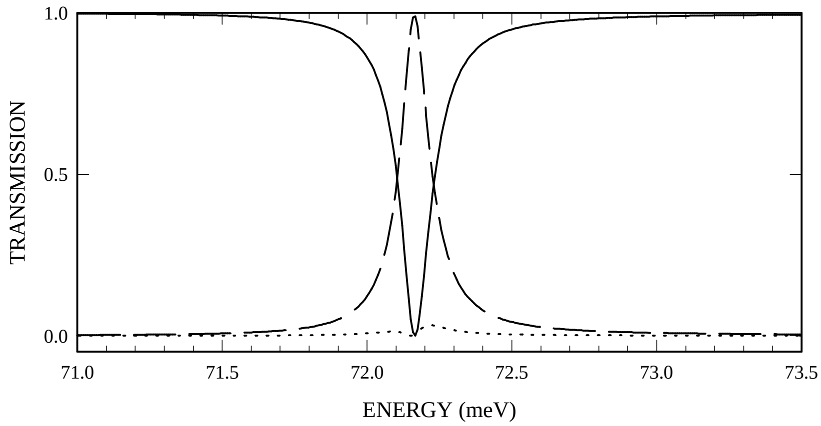

Figure 2. Variation in intensity as a function of energy of the transmitted signal from site 1 to site 2 (solid curve), and the forward signal (T_{13}) (dashed curve), for the structure shown in figure 1(b). The dotted curve represents the signal intensity in the backward direction (T_{14}). These theoretical results were obtained for $N = 6$, $N' = 2$, $d_0 = 16.4$ nm, $d_1 = d_2 = 10$ nm, $d_3 = 16.3$ nm, $d_4 = 32.9$ nm and $m = 0.067m_0$. The resonant energy is $E_0 = E_F = 72.166$ meV.

which define the distances d_3 and d_4 for a given E_0.

In order to illustrate the results of the above analytic theory, we give in figure 2 the variations of the transmission coefficients T_{12}, T_{13} and T_{14} versus the energy E. Figure 2 shows both the dip (solid curve) in the direct transmission from site 1 to site 2 (T_{12}) and the forward drop (dashed curve) from site 1 to site 3 (T_{13}). The backward transferred signal from site 1 to site 4 (T_{14}) is completely absent over the entire energy range and is represented by the dotted curve in figure 2. This application was done for GaAs, with $N = 6, N' = 2, d_0 = 16.4$ nm, $d_1 = d_2 = 10$ nm, $d_3 = 16.3$ nm, $d_4 = 32.9$ nm and $m = 0.067m_0$ where m_0 is the free electron mass. With these parameters, the energy E_0 is equal to 72.166 meV. One can easily check that equations (20) and (25) are verified with these values. The quality factor of the sharp peaks, defined as the ratio between the central energy and the full width at half maximum, is of the order of 580. A more complete study shows that this quality factor depends strongly on the characteristic lengths as well as on the integers N and N'.

The present work can be compared with references [3, 4]. In these papers, the authors consider the tunnelling between two multimode electron waveguides through a potential barrier. Their tunnelling current oscillations for each given transverse mode have a different nature from those we report here in our monomode model, because we have a different tunnelling mechanism due to one localized state. Their study does not allow, as ours does, for the definition of a complete transfer at a given energy. Let us also mention that similar investigations are underway for electromagnetic waves [10,11].

In summary, we have investigated the tunnelling between two monomode continuums coupled by a monomode structure. Our electronic model system allows a complete analytical study of the conditions for a selective transfer of a coherent propagating state from one continuum to the other, leaving all the other neighbour states unaffected. The energy domain where the channel drop tunnelling occurs only depends on the characteristic lengths of the constituents of the model system. We think that our model system may have many applications, in particular, for an electron directional coupler.

This work is partially supported by Le Conseil Régional Nord-Pas de Calais, France.

References

[1] Van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
[2] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1988 *J. Phys. C: Solid State Phys.* **21** L209
[3] del Alamo J A and Eugster C C 1990 *Appl. Phys. Lett.* **56** 78
[4] Eugster C C, del Alamo J A, Rooks M J and Melloch M R 1994 *Appl. Phys. Lett.* **64** 3157
[5] Vasseur J O, Deymier P A, Dobrzynski L, Djafari-Rouhani B, Akjouj A 1997 *Phys. Rev.* B **55** 10434
[6] Singha Deo P and Jayannavar A M 1994 *Phys. Rev.* B **50** 11629
[7] Jin G J, Wang Z D, Hu A and Jiang S S 1999 *J. Appl. Phys.* **85** 1597
[8] See, for example,
Datta S 1995 *Electronic Transport in Mesoscopic Systems* (Cambridge: Cambridge University Press) p 48
[9] Akjouj A, Dobrzynski L, Djafari-Rouhani B, Vasseur J O and Kushwaha M S 1998 *Europhys. Lett.* **41** 321
[10] Fan S, Villeneuve P R, Joannopoulos J D and Haus H A 1998 *Phys. Rev. Lett.* **80** 960
[11] Dobrzynski L, Djafari-Rouhani B, Akjouj A, Vasseur J O and Zemmouri J 1999 unpublished